When Probability Meets Computation

A workshop honouring Guy Latouche on his retirement

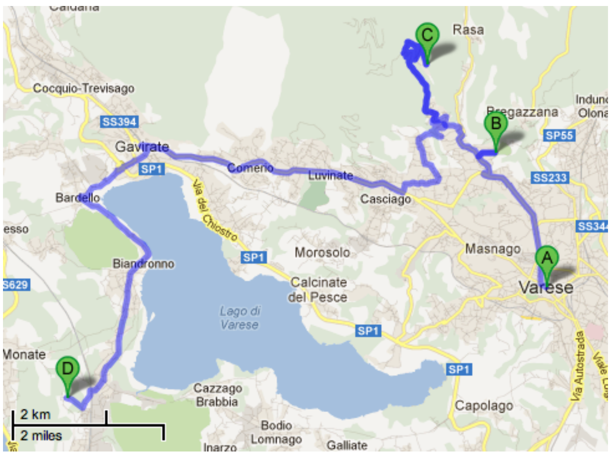
Villa Toeplitz, Varese, Italy, June 6–8, 2012

Practical information

- The workshop takes place in *Villa Toeplitz*, Vico, 46, 21100 Varese.

  From Varese, you can reach the villa by taking the "C" bus in the direction of Sacro Monte. The bus stop is Virgilio, 26 (Prealpi), about 200m from the Villa. The ride takes approximately 15 minutes.

- The excursion is to Sacro Monte di Varese.
- The conference dinner is at *La Voliera Reale*, Via Pacit, 30, 21020 Ternate, Lago Maggiore, Varese.



A: Varese B: Villa Toeplitz
C: Sacro Monte di Varese D: *La Voliera Reale*

Program

Wednesday, June 6

9:20 – 9:30	*Opening*
9:30 – 10:40	Chair: Masakiyo Miyazawa • Nigel Bean: *Spatially-coherent uniformization of a stochastic fluid model to a quasi-birth-and-death process* • Beatrice Meini: *Algebraic Riccati equations associated with M-matrices: theoretical results and algorithms*
	Break
11:00 – 12:10	Chair: Alexander N. Dudin • Sophie Hautphenne: *Extinction probabilities of branching processes with infinitely many types* • Miklos Telek: *Extension of some MAP results to transient MAPs and Markovian binary trees*
	Lunch
14:00 – 15:10	Chair: Nigel Bean • David Meisch: *Parameter estimation via the EM algorithm for a subclass of MVPH* • Giang Nguyen: *Phase-Type Poisson distributions as an extension of Phase-Type distributions*
	Break
15:30 – 16:40	Chair: Miklos Telek • Maria Govorun: *Valuation of long-term medical contracts* • Federico Poloni: *Model estimation through matrix equations in financial econometrics*
17:00 – 19:00	*Welcome Reception (in Villa Toeplitz)*

Thursday, June 7

9:00 – 10:10	Chair: Peter Taylor • Bo Friis Nielsen: *On the relationship between classes of multivariate distributions with rational moment generating function* • Peter Buchholz: *Numerical analysis of rational processes beyond Markov chains*
	Break
10:30 – 12:15	Chair: Dario Bini • Yuanyuan Liu: *Poisson's equation for discrete-time quasi-birth-death processes* • Muhsin Can Orhan: *On the numerical solution of Kronecker-based infinite level-dependent QBDs* • Peter Taylor: *Another look at level-phase independence in GI/M/1 type Markov chains*
	Lunch
14:00 – 15:10	Chair: Attahiru Sule Alfa • Luz Judith R. Esparza: *On size-biased discrete phase-type distributions* • Mogens Bladt: *The estimation of discretely observed Markov jump processes and phase-type distributions*
	Break
15:30 – 18:30	*Excursion to Sacro Monte di Varese*
19:00 – 22:00	*Conference Dinner*

Friday, June 8

9:30 – 10:40	Chair: Udo Krieger • Qi-Ming He: *Optimal (r, Q) policy for an inventory-production system* • Alexander N. Dudin: *Optimization of guard channel policy in cellular mobile networks with account of retrials*
	Break
11:00 – 12:10	Chair: Qi-Ming He • Attahiru Sule Alfa: *Some useful results for the MAP/PH/1 system with PH vacations* • Valeriy Naumov: *Matrix generalization of Erlang's loss formula and its properties*
	Lunch
14:00 – 15:10	Chair: Guy Latouche • Malgorzata O'Reilly: *Loss rates for stochastic fluid models* • Masakiyo Miyazawa: *Markov modulated reflecting fluid process on a multidimensional orthant: Stability and rough asymptotics of the stationary distribution*
	Closing

Abstracts

▶ **Wednesday, 9:30**

Spatially-coherent Uniformization of a Stochastic Fluid Model to a Quasi-Birth-and-Death Process[1]

Nigel G. Bean*, Małgorzata M. O'Reilly

The first paper that used matrix-analytic methods to consider a Markovian stochastic fluid model (SFM) [3], proposed one way of mapping a SFM to a Quasi-Birth-and-Death process (QBD). However, that mapping was not spatially coherent (in the sense that the level of the QBD directly represented the level of the SFM). Instead, it was derived with the specific purpose of allowing algorithmic evaluation of a key matrix (Ψ) using known QBD techniques. In later work, [1],this mapping was further developed in such a way that the *workload* in the queue (not the level of the queue) represented the level of the SFM in an appropriate limit.

In this paper we provide a natural mapping that is spatially coherent since the continuous level in the SFM has a natural correspondence to the discrete level in the QBD process, and so the QBD can be used as a direct approximation of the original SFM. We treat the unbounded as well as bounded cases and illustrate the theory with numerical examples.

The significance of these results goes beyond the development of new ways of approximating the performance measures of a SFM. In particular, we are currently extending these results to the discretization of the driving fluid in a stochastic fluid-fluid model [2]. This will allow for efficient matrix-analytic methods based analysis of this important, but difficult to treat, class of models.

Let $\{(\varphi(t)), t \geq 0\}$ be an irreducible continuous-time Markov Chain (CTMC) with a finite state space $\mathcal{S} = \{1, 2, \ldots, n\}$ and infinitesimal generator $\mathbf{T} = [T_{i,j}]$. Let $\{(\varphi(t), X(t)), t \geq 0\}$ be an SFM with phase variable $\varphi(t)$, level variable $X(t)$ and real rates c_i for all $i \in \mathcal{S}$. Assume initially that $X(t) \in (-\infty, +\infty)$ so that when $\varphi(t) = i$, then the rate at which the level is changing is always c_i.

For a given positive Δx, we introduce discrete levels $k = 0, \pm 1, \pm 2, \ldots$ corresponding to the continuous-level intervals $[k\Delta x, (k+1)\Delta x)$ in the original SFM. For all $i \in \mathcal{S}$, let $\vartheta_i(\Delta x) = \dfrac{|c_i|}{\Delta x}$.

Now we construct the spatially coherent discrete-time QBD $\{(\bar{\varphi}_{\Delta x}(n), \bar{X}_{\Delta x}(n)), n \geq 0\}$ by connecting the phase and level of this QBD to the state of the SFM at arrival times of a Poisson process with an appropriately chosen parameter, following the standard uniformization approach [4]. Choose $\gamma(\Delta x) \geq \max_i \{\lambda_i + \vartheta_i(\Delta x)\}$ and consider the Poisson process $\{N(t), t \geq 0\}$ with parameter $\gamma(\Delta x)$.

The initial condition is $(\bar{\varphi}_{\Delta x}(0), \bar{X}_{\Delta x}(0)) = (\varphi(0), \lfloor \frac{X(0)}{\Delta x} \rfloor)$. Then, by the standard theory of Markov Chains, for example [4], assuming that an arrival in $N(\cdot)$ occurs at time t and that $(\bar{\varphi}_{\Delta x}(N(t-)), \bar{X}_{\Delta x}(N(t-))) = (i, k)$, then

[1] This research is supported by the Australian Research Council through Discovery Project DP110101663.

- with probability $p_1 = \lambda_i/\gamma(\Delta x)$ it is marked as a type-1 arrival and the process moves to some phase $j \neq i$ without a change in discrete level so that a transition from (i,k) to (j,k) occurs (where $j \neq i$ is chosen with probability $T_{i,j}/\lambda_i$),
- with probability $p_2 = \vartheta_i(\Delta x)/\gamma(\Delta x)$ it is marked as a type-2 arrival and the process does not change phase but changes level so that a transition from (i,k) to $(i,k+1)$ occurs if $c_i > 0$, or a transition from (i,k) to $(i,k-1)$ occurs if $c_i < 0$, or
- with probability $p_3 = 1 - p_1 - p_2$ it is marked as a type-3 arrival and the process remains in state (i,k).

This construction yields a discrete-time QBD, but by incorporating the above Poisson process $\{N(t), t \geq 0\}$ directly back into it, we develop the spatially-coherent continuous-time QBD $\{(\varphi_{\Delta x}(t), X_{\Delta x}(t)), t \geq 0\}$.

We formalize this idea in our paper and show that the transition function of the SFM (at the resolution of Δx) can be calculated from that of the continuous-time QBD following an argument similar to the usual uniformization argument. Hence, this QBD is a good approximation for the SFM and captures all the basic dynamics correctly, up to the resolution allowed by the particular choice of Δx. We then show that, in the limit as $\Delta x \to 0^+$, the statistical properties of such a QBD converge to the statistical properties of the SFM and that the key matrix Ψ also emerges from the QBD calculations in this limit.

References

1. S. Ahn and V. Ramaswami. Transient analysis of fluid flow models via stochastic coupling to a queue. Stochastic Models, 20(1):71–101, 2004.
2. N. G. Bean and M. M. O'Reilly. A stochastic fluid model driven by an uncountable-state process, which is a stochastic fluid model itself: Stochastic fluid-fluid model. Submitted, 2012.
3. V. Ramaswami. Matrix analytic methods for stochastic fluid flows. In Pro- ceedings of the 16th International Teletraffic Congress, pages 1019–1030, 1999.
4. S. M. Ross. Introduction to Probability Models. Elsevier, New York, 2007.

▶ **Wednesday, 10:05**

Algebraic Riccati equations associated with M-matrices: theoretical results and algorithms

Beatrice Meini*

Nonsymmetric algebraic Riccati equations (NARE) are nonlinear matrix equations of the kind $C + XA + DX - XBX = 0$, where the unknown X is an $m \times n$ matrix and A, B, C, D are matrices of appropriate size. We focus the attention on NAREs whose block coefficients are such that the matrix

$$M = \begin{bmatrix} A & -B \\ C & D \end{bmatrix}$$

is either a nonsingular M-matrix, or a singular irreducible M-matrix. This class of equations arises in a large number applications, ranging from fluid queues models to transport theory. The solution of interest is the minimal nonnegative one, i.e., the nonnegative solution $X_{\min}$ such that $X_{\min} \leq X$ for any other nonnegative solution X. In this talk we present theoretical properties of the NARE and several numerical methods for the computation of $X_{\min}$. Particular emphasis is given to the properties of the invariant subspaces, and to the techniques used to trasform the eigenvalues of a pencil, keeping unchanged the invariant subspaces. Concerning numerical methods, special attention is addressed to structure-preserving iterative algorithms, having quadratic convergence; connections between the cyclic reduction algorithm and the structure-preserving doubling algorithm (SDA) are pointed out.

▶ **Wednesday, 11:00**

Extinction probabilities of branching processes with infinitely many types

Sophie Hautphenne*, Guy Latouche and Giang Nguyen

In this work, we consider multitype branching processes with infinitely many types and we investigate algorithmic methods to compute the infinite vector of conditional extinction probability given the type of the initial particle. Drawing our inspiration from matrix analytic methods, we propose several converging sequences with probabilistic interpretation, and we discuss the limit of these sequences: some of them converge to the extinction probability of the process, while one converges to another quantity which we interpret as the probability that all types of particles eventually become extinct.

We also discuss extinction criteria, which bring into play the convergence norm of the infinite mean progeny matrix.

▶ **Wednesday, 11:35**

Extension of some MAP results to transient MAPs and Markovian binary trees

Sophie Hautphenne and Miklos Telek*

In this work we extend previous results on moment-based characterization of stationary Markovian Arrival Processes (MAPs) (Bodrog, Horváth, and Telek [3]), and minimal representation of Rational Arrival processes (RAPs) (Buchholz and Telek [4]) to transient Markovian Arrival Processes (TMAPs) and Markovian binary trees (MBTs).

We show that the number of moments that characterize a non-redundant TMAP of size n is n^2+2n-1, and a non-redundant MBT of size n is n^3+2n-1. We provide a non-Markovian representation for both processes based on these moments. Note that any non Markovian representation can be transformed into a Markovian representation by adapting the algorithm developed for stationary MAPs in Telek and Horváth [1].

Next, we discuss the minimal representation of TMAPs and MBTs. In both cases, the minimal representation, which is not necessarily Markovian, can be found using different adaptations of the STAIRCASE algorithm presented in [4].

Finally, we investigate canonical representations for TMAPs and MBTs of order 2. Different Markovian canonical forms exist for both processes, but to the best of our knowlegde, a formal classification of TMAPs(2) and MBTs(2) according to their canonical form(s) seems even harder to find than in the stationary MAP(2) case (Bodrog *et al.* [2]).

References

1. M. Telek and G. Horváth. A minimal representation of markov arrival processes and a moments matching method. *Performance Evaluation*, 64(9-12):1153–1168, Aug. 2007.

2. L. Bodrog, A. Heindl, G. Horváth, and M. Telek, A markovian canonical form of second-order matrix-exponential processes, *European Journal of Operation Research*, vol. 190, pp. 459–477, 2008

3. L. Bodrog, A. Horváth, and M. Telek. Moment characterization of matrix exponential and Markovian arrival processes. *Annals of Operations Research*, 160:51–68, 2008.

4. P. Buchholz and M. Telek. On minimal representation of rational arrival processes. *Annals of Operations Research*, 2011. DOI 10.1007/s10479-011-1001-5.

▶ **Wednesday, 14:00**

Parameter estimation via the EM algorithm for a subclass of MVPH

Mogend Bladt, David Meisch* and Bo Friis Nielsen

The Expectation-Maximization (EM) algorithm is well-known for fitting phase-type distributions to given univariate data. In several fields the information received from data is often multivariate and correlated. Our goal is to fit a subclass of multivariate phase-type distributions (MVPH) to given multivariate data. A first approach is to fit a bivariate exponential distribution, more specifically a Kibble distribution using an adaption of the EM algorithm.

A Kibble distribution (also known as Jensen's, Gaver's or Downton-Moran's distribution), is a bivariate exponential distribution. It can be used to model two components receiving shocks occurring in independent Poisson processes which need a bivariate geometric distributed number of shocks until failure of the components. This has e.g. an application in reliability theory.

Kotz, Balakrishnan and Johnson list several methods proposed by various authors to estimate the parameter of this distribution in their book "Continuous Multivariate Distributions". Additional approaches have been published. To our knowledge it has not been previously tried to use an EM algorithm to estimate the parameters of a Kibble distribution.

In order to determine the performance of our EM algorithm, we have calculated parameter estimates using the EM algorithm as well as several other methods using simulated data, saving all relevant information and comparing the estimates to our original generator and the simulated data. A well-known characteristic of the EM algorithm is that it always converges, and in this special case the calculation of the estimates converges exponentially.

▶ **Wednesday, 14:35**

Phase-Type Poisson distributions as an extension of Phase-Type distributions

Sophie Hautphenne, Guy Latouche and Giang Nguyen*

Phase-Type Poisson distributions may be examined from different perspectives. *Matrix-form Poisson distributions* were first introduced in Wu and Li [2] as one generalization of Panjer distributions, and subsequently analyzedin Siaw *et al.* [1]. The main focus of these two papers was to extend Panjer's algorithm — an efficient recursive procedure for evaluating compound distributions, assuming that the number of summands follows a Panjer distribution—to more general families of distributions, including the Matrix-form Poisson distributions.

Here, we show that when certain positivity constraints are satisfied, these Matrix-form Poisson distributions have a physical interpretation as extensions of Phase-Type distributions. We refer to these positive Matrix-form Poisson distributions as *Phase-Type Poisson distributions*, and use our physical interpretation to construct an EM algorithm for parameter estimation.

References

1. K. K. Siaw, X. Wu, D. Pitt, and Y. Wang. Matrix-form recursive evaluation of the aggregate claims distribution revisited. *Anals of Actuarial Science*, 5(2):163–179, 2011.

2. X. Wu and S. Li. Matrix-form recursions for a family of compound distributions. *ASTIN Bulletin*, 40:351-368, 2010

▶ Wednesday, 15:30

Valuation of long-term medical contracts

Maria Govorun*, Guy Latouche and Stephane Loisel

In the present work we use phase-type methods to examine the discounted future losses of a long-term medical care contract.

Many life insurance companies are exposed to risks related to long-term medical contracts. The management of these risks is a complicated problem because it involves questions of a long-and a short-term perspective. For a company selling such a contract, the long perspective question would be to correctly estimate the present value of all its potential losses. In a short term perspective the company needs to satisfy regulatory requirements that usually impose restrictions on a short-time basis, which means that the risks have to be calculated with a good precision at any time.

For a long-term horizon we define two discrete time models for the present value that are based on different assumptions for the treatment costs. In the first model, the treatment costs of each year of life are independent and identically distributed random variables. Thus, they do not depend on aging of the individual. In our second model, we assume that the treatment costs depend on the health state of the individual and they are defined by a Markov reward process.

In order to meet regulatory requirements on a short term horizon we develop a continuous time model, where the present value of losses is described by a fluid queue.

For the remaining lifetime of an individual we use a continuous phase-type model introduced by Lin and Liu in 2007, where a Markov chain is used to model human mortality and where the phases represent health states of individuals. This enables us to obtain algorithmic procedures to compute the density and the distribution function of the present value.

We use the models to perform different stress tests. For instance, tests with respect to mortality rates allow us to study the impact of an increased lifetime spent in bad health states for which medical treatments are the most expensive.

▶ **Wednesday, 16:05**

Model estimation through matrix equations in financial econometrics

Federico Poloni* and Giacomo Sbrana

I would like to present a recent project involving applied probability and matrix equation in a novel way.

The GARCH(1,1) process [Engle, '82, Bollerslev, '86] is a stochastic process that models the conditional (co)variance of financial time series. Namely, a GARCH(1,1) is a sequence of the form $y_t = H_t^{1/2}\epsilon_t$, $t = 1, 2, \dots, T$, where the $\epsilon_t \in \mathbb{R}^d$ are i.i.d. with mean 0 and variance I and the conditional variance $H_t \in \mathbb{R}^{d\times d}$ is given by a linear affine function of H_{t-1} and $y_{t-1}y_{t-1}^T$.

Estimating the parameters of this affine function from a number of observations of the vector y_t is a challenging task, since the data essentially consist of "noise" only. For this purpose, the standard choice is a maximum-likelihood estimator obtained through a general-purpose quasi-Newton optimization technique. This method is often computationally intensive and slow to converge, especially when d increases.

Generalizing an approach by [Linton, Kristensen '06] for the univariate (scalar) GARCH(1,1), we suggest a new estimation strategy for the GARCH(1,1) parameters using the solution of a palindromic matrix equation $\Gamma_1 X^2 + \Gamma_0 X + \Gamma_1^T$; here, $\Gamma_0 = \Gamma_0^T$ and Γ_1 are $d(d+1)/2 \times d(d+1)/2$ matrices depending on the moments of y_t only, and can thus be estimated easily from the observations.

While this estimator is not as accurate as the maximum-likelihood one, it is much faster to compute and can be refined using a simple fixed-point iteration.

▶ **Thursday, 9:00**

On the relationship between classes of multivariate distributions with rational moment generating function.

Mogend Bladt and Bo Friis Nielsen*

The class of multivariate distributions with rational moment generating function contains an important sub-class of distributions that can be interpreted as different linear rewards earned under the sojourns in the transient states of a finite terminating Markov chain. Until now it has been an open question whether there exist distributions with rational moment generating function that do not belong to this subclass. We provide an example settling that at least in the case of general real rewards the class is a strict sub-class.

▶ **Thursday, 9:35**

Numerical Analysis of Rational Processes Beyond Markov Chains

Peter Buchholz*

If one skips the probabilistic interpretation of transitions in PH distributions and MAPs the more general classes of ME distributions [6] and RAPs [1] can be defined. Although, these models are known for some time they have rarely been used in stochastic models since it was unclear whether the resulting models are valid stochastic process and if they are stochastic processes can be analyzed and it is also unclear whether a given set of matrices and vectors describe a valid process. Only recently work has been started towards a more general use of these models in a larger setting. In particular it has been shown that PH distributions and MAPs have an up to similarity transformations unique minimal representation as an ME distribution or RAP [4] and that ME distributions and RAPs can be used in stochastic models like SPNs [3] or SANs [5] under some restrictions and a valid stochastic process denotes as a rational process (RP) results from the model which in principle can be analyzed numerically. Furthermore, it has been proved that matrix analytic methods are still valid and applicable if ME/RAPs are used instead of PH/MAPs [2].

Even if it is known that the analysis of RPs is in principle similar to the analysis of Markov processes, many details are open. Due to the missing stochastic interpretation in RPs, the rates between states or values in a stationary or transient state vector can be negative in RPs which for example implies that established numerical methods for Markov processes like uniformization for transient analysis or SOR for stationary analysis do not work for RPs.

In the talk we give an overview of RPs and introduce first computational methods to compute the stationary state vector of RPs with finite state spaces. From the stationary state vector quantitative results like sojourn times or throughputs can be derived. It is shown that by exploiting the model structure of the model the state space of an RP can be decomposed such that the sum of elements in a subset is a probability and the values over all subsets form a probability distribution. According to the decomposition of the state space a block structure on the transition matrix with non-singular diagonal blocks can be defined. This and some other features of the decomposition can be exploited to define computational methods for RPs. The talk introduces some of those methods and some experimental results indicating that the use of RPs rather than Markov processes can be beneficial in several situations. Furthermore, it lists some open research problems in the area.

References

1. S. Asmussen, M. Bladt. Point processes with finite-dimensional conditional probabilities. Stochastic Processes and Their Application 82 (1999), 127-142.

2. N. Bean, B. F. Nielsen. Quasi-birth-death processes with rational arrival process components. Stochastic Models 26 (3), 2010, 309-334.

3. P. Buchholz, M. Telek. Stochastic Petri Nets with Matrix Exponential firing times. Performance Evaluation 67 (12), 2010, 1373-1385.
4. P. Buchholz, M. Telek. Rational processes related to communicating Markov processes. Journal of Applied Probability 49 (1), 2012, 40-59.
5. P. Buchholz, M. Telek. Rational automata networks. INFORMS Journal on Computing (to appear).
6. L. Lipsky. Queueing Theory. Springer 2008.

▶ **Thursday, 10:30**

Poisson's equation for discrete-time quasi-birth-death processes

Sarah Dendievel, Guy Latouche and Yuanyuan Liu*

In this talk, we will present our recent our results on Poisson's equation for positive recurrent, irreducible and aperiodic discrete-time QBD processes with finite phases. The solutions of Poisson's equation are expressed explicitly in terms of both the functionals of the first hitting times and the deviation matrix. The link between the solution of Poisson's equation and central limit theorem, perturbation analysis and measure of convergence rates is also investigated.

▶ **Thursday, 11:05**

On the numerical solution of Kronecker-based infinite level-dependent QBDs

Tuğrul Dayar and Muhsin Can Orhan*

Infinite level-dependent quasi-birth-and-death processes (LDQBDs) can be used to model Markovian systems with countably infinite multi-dimensional state spaces. Recently it has been shown that sums of Kronecker products can be used to represent the nonzero blocks of the infinitesimal generator matrix underlying an LDQBD. Then the challenge in the matrix analytical solution is to compute conditional expected sojourn time matrices of the LDQBD under low memory and time requirements. In this talk, we present results of numerical experiments with a Kronecker-based matrix-analytic solution method on systems having more than two countably infinite dimensions modeled as LDQBDs and derive various rules of thumb.

References

1. L. Bright and P.G. Taylor. Calculating the equilibrium distribution in level dependent quasi- birth-and-death processes. Stochastic Models, 11:497–525, 1995.

2. T. Dayar and M.C. Orhan. LDQBD solver version 2. http://www.cs.bilkent.edu.tr/ tugrul/software.html, 2011.

3. T. Dayar, W. Sandmann, D. Spieler, and V. Wolf. Infinite level-dependent QBD processes and matrix analytic solutions for stochastic chemical kinetics. Advances in Applied Probability, 38:1005–1026, 2011.

4. G. Latouche and V. Ramaswami. Introduction to Matrix Analytic Methods in Stochastic Modeling. SIAM Press, Philadelphia, PA, 1999.

5. M.F. Neuts. Matrix-Geometric Solutions in Stochastic Models. The Johns Hopkins University Press, Baltimore, MD, 1981.

6. M.C. Orhan. Kronecker-based infinite level-dependent QBDs: Matrix analytic solution versus simulation. Master's thesis, Department of Computer Engineering, Bilkent University, 2011.

7. V. Ramaswami and P.G. Taylor. Some properties of the rate operators in level depen- dent quasi-birth-and-death processes with a countable number of phases. Stochastic Models, 12:143–164, 1996.

▶ **Thursday, 11:40**

Another look at level-phase independence in $GI/M/1$ type Markov chains

Guy Latouche, Safieh Mahmoodi and Peter G. Taylor*

It is well-known that, with the stationary distribution $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \ldots)$ partitioned according to the levels, a positive-recurrent $GI/M/1$ type Markov chain with finitely-many phases and generator of the form

$$P = \begin{bmatrix} B_0 & A_1 & 0 & 0 & \cdots \\ B_{-1} & A_0 & A_1 & 0 & \cdots \\ B_{-2} & A_{-1} & A_0 & A_1 & \ddots \\ B_{-3} & A_{-2} & A_{-1} & A_0 & \ddots \\ \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix},$$

has a matrix-geometric stationary distribution defined by $\boldsymbol{\pi}_k = \boldsymbol{\pi}_0 \widehat{R}^k$, with $\widehat{R}$ the minimal nonnegative solution of the matrix equation

$$R = \sum_{k=0}^{\infty} R^k A_{-k+1},$$

and where

$$\boldsymbol{\pi}_0 = \boldsymbol{\pi}_0 \sum_{k=0}^{\infty} \widehat{R}^k B_{-k},$$

with

$$\boldsymbol{\pi}_0 \sum_{k=0}^{\infty} \widehat{R}^k \mathbf{1} = 1.$$

In [1] we took the matrix sequence $\{A_{-k}\}$ as given, and showed how to construct a sequence of boundary matrices $\{B_{-k}\}$, so that the stationary distribution is level-phase independent, that is $\boldsymbol{\pi}_k = (1-\rho)\rho^k \boldsymbol{u}$ for some suitable $\rho \in [0,1]$ and $\boldsymbol{u}$. The proof there involved studying the doubly-infinite Markov chain with generator

$$T = \begin{bmatrix} \ddots & \ddots & \ddots & & & \\ \ddots & A_0 & A_1 & 0 & & \\ & A_{-1} & A_0 & A_1 & 0 & \\ & A_{-2} & A_{-1} & A_0 & A_1 & \ddots \\ & & \ddots & \ddots & \ddots & \ddots \end{bmatrix}, \tag{1}$$

censored so that it stays in the nonnegative levels. An adjustment, with an interesting physical interpretation, had to be made to ensure that the resulting matrix is conservative.

More recently, we have been asking the same question about $GI/M/1$ type Markov chains with infinitely-many phases. For such chains, there can be a continuum of scalars $\rho \in [0,1]$ and $\boldsymbol{u} \in \ell_1$ for which $\boldsymbol{u}R = \rho\boldsymbol{u}$, and for each of these, we can employ techniques, similar to those described above, to show that boundary transitions can be constructed, so that the stationary distribution of the chain is level phase independent with $\boldsymbol{\pi}_k = (1-\rho)\rho^k\boldsymbol{u}$.

References

1. G. Latouche and P.G. Taylor, Level-Phase Independence in Processes of Gi/M/1 Type, *Journal of Applied Probability*, 37, 984?998, 2000.

▶ **Thursday, 14:00**

On size-biased discrete phase–type distributions

Mogend Bladt, Luz Judith R. Esparza* and Bo Friis Nielsen

Observations of certain phenomena may suffer from bias caused by the sampling method by which the data were collected. Situations leading to discrete weighted distributions include the analysis of family data, the aerial survey involving visibility bias in wildlife ecology, line transect sampling, among others. In this paper, we analyze some models leading to size-biased discrete phase–type distributions and matrix–geometric distributions, showing that factorial moment distributions of these classes remain within their respective classes. We give explicit discrete phase–type and matrix–geometric representations of these factorial moment distributions.

A special and important case is the first factorial moment distribution, where the probability of selection is proportional to the size of the observation. A well known situation applying this special case is the inspection paradox from discrete-time renewal processes.

We discuss the possibility of estimating general moment phase–type distributions via the EM algorithm. Applications with real data on family size and grain size-distributions using this algorithm are considered.

▶ **Thursday, 14:35**

The estimation of discretely observed Markov jump processes and phase-type distributions

Mogens Bladt*, Bo Friis Nielsen and Luz Judith R. Esparza

We consider one or several independent Markov jump processes of finite state-space which are observed at discrete times only. We are concerned with the estimation of the intensity matrix of the underlying continuous process given this partial information. Classical approaches for dealing with incomplete information like the EM algorithm and Markov chain Monte Carlo methods will be employed. In the special case where the Markov jump processes generate a phase-type distribution, the method is combined with well-known methods for phase-type estimation. We present a real example from the world of credit risk rating and a simulated example from survival analysis. Possible extensions to Markov modulation of the intensities over time will also be considered.

▶ **Friday, 9:30**

Optimal (r, Q) Policy for an Inventory-Production System

Qi-Ming He* and Hanqin Zhang

We consider a system consisting of a warehouse and a production facility. An (r, Q) policy is used for inventory management in the warehouse. A quantity policy is used for shipment consolidation in the production facility. More specifically, the inventory-production system (see the figure) is defined as follows.

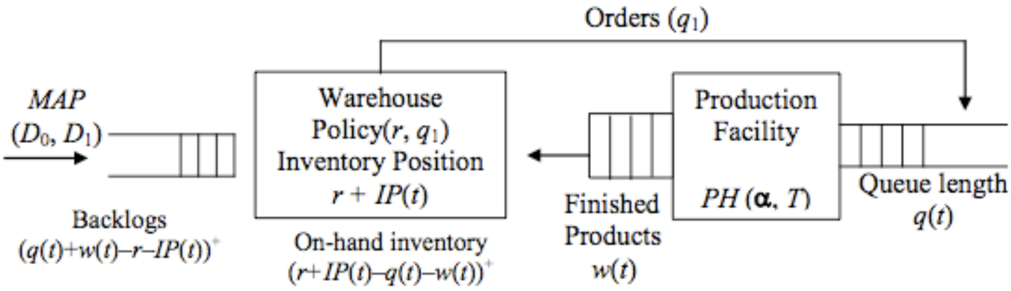
1. Customer demands arrive according to a Markov arrival process (MAP) with a matrix representation (D_0, D_1).

2. Inventory in the warehouse is reviewed continuously. The warehouse adopts an (r, q_1) policy for its inventory management. That is: whenever the inventory position reaches r, an order of the amount q_1 is placed to the production facility.

3. The production facility always has enough resource for production. The production facility produces one product at a time. The production time of a product has a phase-type distribution with a PH-representation $(\boldsymbol{\alpha}, T)$.

4. Finished products are stored in the production facility first. Once the number of finished products reaches q_2, the whole batch of finished products is shipped together to the warehouse. Note that q_2 is a positive integer.

5. The holding cost per product in the warehouse per unit time is h_w, penalty cost per demand per unit time in the warehouse is p_w, the ordering cost per order in the warehouse is K, and the holding cost per product in the production facility per unit time is h_s.



By using the matrix-analytic methods, an algorithm is developed for computing the expected system costs. We also explore methods for finding the policy for inventory management in the warehouse and shipment management in the production facility that minimizes the expected system costs. A highlight of the research is the explicit result on the shipment consolidation at the production facility.

▶ **Friday, 10:05**

Optimization of Guard Channel Policy in Cellular Mobile Networks with Account of Retrials

Valentina I. Klimenok, Alexander N. Dudin* and Chesoong Kim

Importance of the problem of optimal handling the handover calls in wireless networks is well-recognized. Small number of channels in a cell of the network and the competition between calls may create essential problems, especially for moving users. From a user's perspective, it is more intolerable to drop an on-going service, than to block a service that has yet to be established. Therefore, with limited bandwidth in a cell, satisfying requests of on-going (handover) calls is more important than satisfying the requests of the new calls generating in a given cell. Thus, different policies that should provide some kind of priority to handover calls over the new calls are elaborated. Well known is so called Guard Channel Policy which assumes reservation of some part of channels for service of the handover calls. Under such a policy, optimization problem arises: how many servers should be reserved exclusively for service of the handover calls. There is a lot of works where such type of optimization problems has been considered. The main shortcomings of that considerations are the following imposed assumptions: (i) arrival flows of the handover and new customers are defined as independent stationary Poisson arrival processes while arrival flows in modern wireless mobile communication networks exhibit correlation and high variability of inter-arrival times; (ii) retrials are absent or retrial rate from the orbit is constant; (iii) service times of the handover and new customers are identically distributed; ($iiii$) system is a priori stable (ergodic).

In the present work, we consider the model of Guard Channel Policy that is free of the indicated shortcomings of previous considerations. To this end, we consider the $(N + R)$-server queueing system without waiting space. Customers of two types arrive to the system according to the *Marked Markovian Arrival Process*. We interpret type-1 customers as hand-over customers and type-2 customers as new customers generated in the given cell. We assume that handover customers have a priority. This priority is provided by means of reservation of R servers especially for type-1 customers. Arriving type-1 customer is rejected (dropped) only if all $N + R$ servers are busy. Arriving type-2 customer is blocked only if at least N servers are busy at its arrival moment. This customer leaves the system with probability $1 - p$, $0 \leq p \leq 1$. With complementary probability, p, type-2 customers moves the orbit of infinite capacity and retries for the service later on. Each customer staying in the orbit makes retrials independently of other customers. Inter-retrial time has exponential distribution with parameter η. If the number of busy servers at a retrial moment is less than N, the customer occupies a free server and starts a service. In the opposite case, with probability q, $0 \leq q \leq 1$, the customer returns to the orbit or, with probability $1 - q$, leaves the system (is lost). Service of type-k customer has exponential distribution with parameter $\mu_k,\ k = 1, 2$.

Using matrix analytic technique we derive the non-trivial ergodicity condition, calculate stationary distribution and the main performance measures of the system. The optimization issues are discussed.

▶ **Friday, 11:00**

Some useful results for the MAP/PH/1 system with PH vacations

Attahiru Sule Alfa*

Polling systems which occur frequently in the medium access control aspects in communication systems are usually approximated by vacation queueing models. For the MAP/PH/1 system with PH vacations the model can be set up as QBD. Sometimes the length and type of vacation models considered lead to huge sizes of the associated matrices R and G. With computational focus in mind we present decomposition results associated with these matrices. We show that most of the work in computing these matrices is simply that of computing the equivalent matrices for the MAP/PH/1 system (without vacation), and that the remaining work only involves solving linear equations. By being able to decompose the matrices and then capitalizing on the features of this decomposition we can considerably reduce the associated computational efforts. We also show that these matrices can be obtained explicitly for the Geo/PH/1 with PH type vacation. This is an extension of the well known results by Ramaswami and Latouche (1986) for the Geo/PH/1 without vacation.

▶ **Friday, 11:35**

Matrix generalization of Erlang's loss formula and its properties

Valeriy Naumov*

We consider loss system MAP/M/n with n servers and multi-class Markovian Arrival Process. Arrival process is specified by irreducible generator matrix Q and non-negative matrices $R_1, \ldots, R_m$, where matrix $R = R_1 + \ldots + R_m$ satisfies $R(i,j) \leq Q(i,j)$ for $i \neq j$. Matrices $S = Q - R$ and R_k define arrival process of class k calls (Latouche and Ramaswami, 1999). Service times of all calls are exponentially distributed with parameter 1. Let $\boldsymbol{p}$ be a row vector of the stationary probabilities of Q, $\mathbf{1}$ be a column vector of all ones, and matrices Ψ_k be defined as

$$\Psi_0 = 0, \quad \Psi_k = \left(I - \frac{1}{k}(S + \Psi_{k-1}R) \right)^{-1}, \ k = 1, 2, \ldots.$$

We use properties of block generator matrices (Naumov, 2012) and show that Ψ_n satisfies the following properties:

1. $\Psi_n \geq 0$;
2. $\Psi_n R\mathbf{1} \leq n\mathbf{1}$;
3. $\boldsymbol{p}\Psi_n \leq \boldsymbol{p}$;
4. Time blocking probability is given by
$$E = \boldsymbol{p}(I - \Psi_n)\mathbf{1}$$
5. Call blocking probability for class k calls is given by
$$B_k = \frac{\boldsymbol{p}(I - \Psi_n)R_k\mathbf{1}}{\boldsymbol{p}R_k\mathbf{1}}.$$

We also derive explicit formulas for the stationary covariance of the number of servers occupied by calls of different classes in the infinite-server system that processes calls lost at the primary system MAP/M/n.

References

1. Latouche, G. and V.Ramaswami. 1999. Introduction to Matrix Analytic Methods in Stochastic Modeling. ASA-SIAM, Philadelphia.

2. Naumov, V. 2012. Matrix-Multiplicative Solutions of Finite Markov Chains. ETLA, Helsinki (forthcoming).

▶ **Friday, 14:00**

Loss Rates for Stochastic Fluid Models[2]

Małgorzata M. O'Reilly* and Zbigniew Palmowski

We introduce loss rates, a novel class of performance measures for stochastic fluid models (SFMs) and discuss their applications potential. We derive analytical expressions for loss rates and describe efficient methods for their evaluation. Further, we study interesting asymptotic properties of loss rates and derive explicit expressions which can be conveniently evaluated even for systems of large size. Numerical examples are given as an illustration.

Let $\{(\varphi(t)), t \geq 0\}$ be an irreducible, positive-recurrent continuous-time Markov Chain (CTMC) with a finite state space $\mathcal{S} = \{1, 2, \ldots, n\}$ and infinitesimal generator $\mathbf{T}$. Let $\{(\varphi(t), X(t)), t \geq 0\}$ be a Markovian stochastic fluid model (SFM) with phase variable $\varphi(t)$, level variable $X(t)$, a lower boundary $X(t) \geq 0$, upper boundary $X(t) \leq B$, and real rates c_i for all $i \in \mathcal{S}$, such that

- When $0 < X(t) < B$ and $\varphi(t) = i$, then the rate at which the level is changing is c_i;

- When $X(t) = 0$ and $\varphi(t) = i$, then the rate at which the level is changing is $\max\{c_i, 0\}$; and

- When $X(t) = B$ and $\varphi(t) = i$, then the rate at which the level is changing is $\min\{c_i, 0\}$.

We partition the set of all phases as $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_0$, where $\mathcal{S}_1 = \{i \in \mathcal{S} : c_i > 0\}$, $\mathcal{S}_2 = \{i \in \mathcal{S} : c_i < 0\}$, $\mathcal{S}_0 = \{i \in \mathcal{S} : c_i = 0\}$.

This class of models has been earlier analyzed by da Silva Soares and Latouche, who derived the stationary distribution of the model in [1]. In this paper, we extend the analysis to the study of other interesting performance measures of the considered model.

Throughout assume $i \in \mathcal{S}_1$ and $j \in \mathcal{S}_2$. We introduce *relative-time loss rates*, a measure of time spent at the boundary B with respect to the duration of the busy period, as follows. Let $\theta(x) = \inf\{t > 0 : X(t) = x\}$ be the first passage time to level x. Define

$$E^{\theta(0)}_{i,j} = E[\theta(0) : \varphi(\theta(0)) = j | \varphi(0) = i, X(0) = 0],$$

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interpreted as the mean first passage time to level zero and doing so in phase j, given start in level zero and phase i. Let $\tau(t) = \int_u^t I(X(u) = B)du$ be the total time spent on the upper boundary B up to time t. Define

$$E_{i,j}^{\tau(\theta(0))} = E[\tau(\theta(0)) : \varphi(\theta(0)) = j|\varphi(0) = i, X(0) = 0],$$

interpreted as the mean time spent at the boundary B before visiting level zero and doing so in phase j, given start in level zero and phase i. We define relative-time loss rate Z_{ij} by

$$Z_{i,j} = E_{i,j}^{\tau(\theta(0))} / E_{i,j}^{\theta(0)}. \tag{2}$$

We note that no fluid is lost during the times when the process is at the boundary B in some phase k with $c_k = 0$. Consequently, we introduce the *absolute-volume loss rates*, which is a measure of total fluid lost during the busy period. Let $V(t) = \int_0^t c_{\varphi(u)} I(X(u) = B) I(c_{\varphi(u)} > 0)du$. be the total fluid volume lost up to time t. Such loss occurs whenever the buffer X is full and the fluid level is increasing. Define

$$E_{i,j}^{V(\theta(0))} = E[V(\theta(0)) : \varphi(\theta(0)) = j|\varphi(0) = i, X(0) = 0],$$

interpreted as the mean total fluid volume lost at the moment of the first return to level zero and doing so in phase j, given start in level zero and phase i. We define absolute-volume loss rate M_{ij} by

$$M_{i,j} = E_{i,j}^{V(\theta(0))} / E_{i,j}^{\theta(0)}. \tag{3}$$

We also introduce *relative-volume loss rates* which are the measure of the total fluid lost with respect to the total fluid that entered the buffer during the busy period. Let $W(t) = \int_0^t c_{\varphi(u)} I(c_{\varphi(u)} > 0)du$ be the total fluid volume that went *into* buffer X up to time t. Define

$$E_{i,j}^{W(\theta(0))} = E[W(\theta(0)) : \varphi(\theta(0)) = j|\varphi(0) = i, X(0) = 0],$$

interpreted as the mean total fluid that went into buffer X up to the moment of the first return to level zero and doing so in phase j, given start in level zero and phase i. We define relative-volume loss rate M_{ij}^* by

$$M_{i,j}^* = E_{i,j}^{V(\theta(0))} / E_{i,j}^{W(\theta(0))}. \tag{4}$$

We will also analyze the asymptotics of all of the above quantities as B tends to infinity. The goal is to qualify quickly the appropriate size of the maximal buffer size B such that chosen loss rate is smaller than given small threshold describing the performance of the system.

References

1. A. da Silva Soares and G. Latouche. Matrix-analytic methods for fluid queues with finite buffers. Performance Evaluation, 63:295–314, 2006.

▶ **Friday, 14:35**

Markov modulated reflecting fluid process on a multidimensional orthant: Stability and rough asymptotics of the stationary distribution

Masakiyo Miyazawa*

Motivated by queueing network applications, we consider a continuous-time Markov modulated reflecting process on a nonnegative orthant of a multidimensional Euclidean space. This orthant is partitioned into a boundary and an interior, where the boundary is composed of faces which are specified by the coordinates being vanished. This reflecting process is modulated by a Markov chain with finitely many states. We assume that the reflecting process is continuous and piecewise linear in time changing its direction when either it hits the boundary or the background state is changed. We refer to it as a Markov modulated reflecting fluid process on the nonnegative multidimensional orthant, a multidimensional MMRF process for short.

This fluid process is specified by the transition matrix of the background Markov chain and the set of direction vectors, where a direction is uniquely determined by the background state and the current location of the reflecting process with respect to the boundary faces or the interior. Thus, the transition from the boundary may depend on its faces, and may be arbitrarily given.

Our primary interests are in the stability and tail asymptotics of the stationary distribution of the multidimensional MMRF process. These problems are generally very hard to answer because of the multidimensional reflecting structure. This may be the reason why this process has not been well studied except for simple models such as single or tandem fluid queues. Obviously, the process is closely related to a Markov modulated multidimensional reflecting random walk. There has been some fundamental work for the reflecting random walk without Markov modulation in the ninety. In recent years, some new results have been and are going to be obtained for the stability as well as the tail asymptotics. Although we can not directly use them because their reflecting mechanism is slightly different, some ideas are still useful for the multidimensional MMRF process.

This is the first attempt for studying the multidimensional MMRF process, and therefore we scratch it from its definition. We may not be able to go far, but will try to draw a grand design for solving the problems. For this, we have the following plan. We consider a general criterion on the stability of the multidimensional MMRF process. Provided this stability is assumed, we propose a general framework to derive rough asymptotics of the marginal stationary distribution in an arbitrary direction. For this, we will use the convergence domain of the moment generating function of the stationary distribution. We also discuss some examples to see how our results work.

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